

# Phases of QCD

lattice thermodynamics and a field theoretical model

Claudia Ratti

*ECT\*, Trento, ITALY and Technical University, Munich, GERMANY*

Can results of  
**Lattice QCD Thermodynamics**  
be understood in terms of  
**QUASIPARTICLE**  
degrees of freedom?

C. R., M. Thaler, W. Weise, hep-ph/0506234.

P. Meisinger and M. Ogilvie (1996)- K. Fukushima (2004)

## Polyakov loop extended NJL model

**Starting point:** NJL model in temporal background gauge field

$$\mathcal{L}_{PNJL} = \bar{\psi} (i\gamma_\mu D^\mu - \hat{m}_0) \psi + \frac{G}{2} \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5 \vec{\tau}\psi)^2 \right] - V(\Phi, T),$$

where:

$$D_\mu = \partial_\mu + igA_\mu \quad \text{and} \quad A_\mu = \delta_{\mu 0} A_0 .$$

Coupling between Polyakov loop and quarks **uniquely determined** by covariant derivative  $D_\mu$ .  
We recall that:

$$\Phi(x) = \frac{1}{N_c} \text{Tr} \left[ \mathcal{P} \exp \left( i \int_0^\beta A_0(x, \tau) d\tau \right) \right].$$

### Parameters

$\Lambda$ [GeV]	0.651
$G$ [GeV $^{-2}$ ]	10.078
$m_0$ [MeV]	5.5

### Physical quantities

$f_\pi$ [MeV]	92.4
$ \langle \bar{\psi}\psi \rangle ^{1/3}$ [MeV]	247
$m_\pi$ [MeV]	139.3

## Polyakov loop potential

R. Pisarski (2000)

- ◆ The Polyakov loop is the **order parameter** related to deconfinement in **pure gauge QCD**

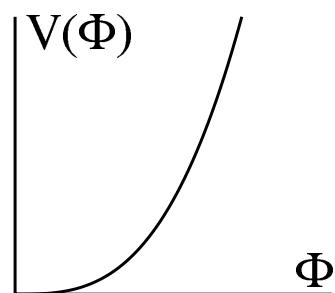
$$\frac{V(\Phi, T)}{T^4} = -\frac{b_2(T)}{2}\Phi^\dagger\Phi - \frac{b_3}{6}\left(\Phi^3 + \Phi^{\dagger 3}\right) + \frac{b_4}{4}\left(\Phi^\dagger\Phi\right)^2$$

with

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

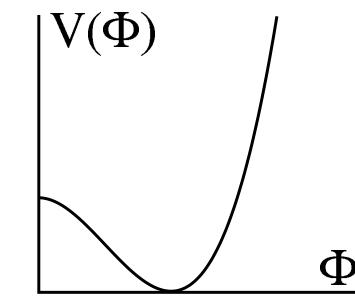
$$T < T_c$$

- color confinement
- $\langle\Phi(\vec{x})\rangle = 0$



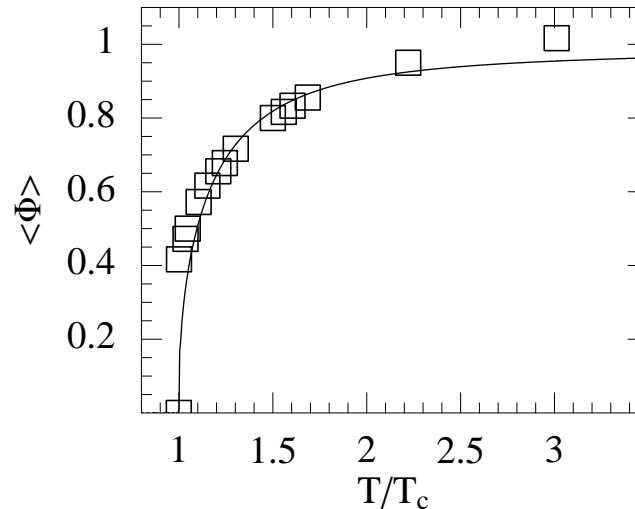
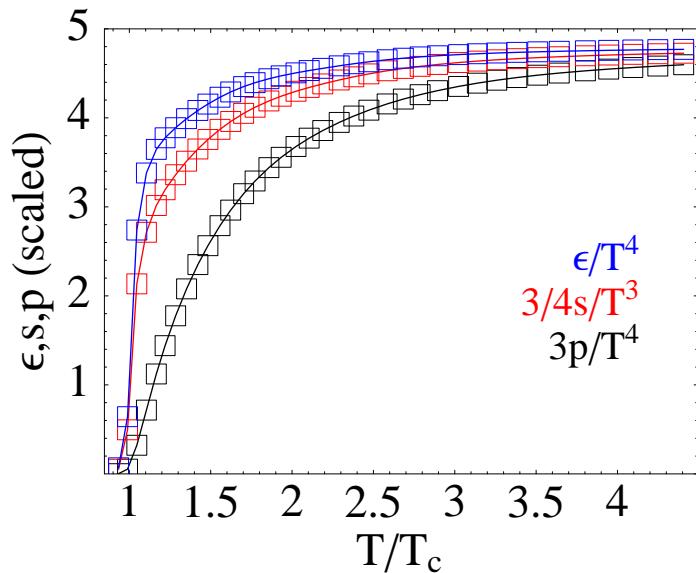
$$T > T_c$$

- color deconfinement
- $\langle\Phi(\vec{x})\rangle \neq 0$



## Comparison with lattice data

- ❖ Minimization of  $V(\Phi, T)$ : Polyakov loop expectation value as a function of  $T$
- ❖ Comparison with lattice data from Kaczmarek *et al.* PLB 543 (2002)



- ❖  $p(T) = -V(\Phi(T), T)$
- ❖  $s(T) = \frac{dp}{dT} = -\frac{dV(\Phi(T), T)}{dT}$
- ❖  $\epsilon(T) = T \frac{dp}{dT} - p = Ts(T) - p(T)$
- ❖ Comparison with lattice data from Boyd *et al.* NPB 469 (1996)

## PNJL model at finite temperature and chemical potential

The thermodynamic potential of the system is:

$$\Omega(T, \mu) = V(\Phi, T) - T \sum_n \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2} \text{Tr} \ln \left( \frac{1}{T} \tilde{S}^{-1}(i\omega_n, \vec{p}) \right) + \frac{\sigma^2}{2G},$$

where  $\omega_n = (2n + 1)\pi T$  are the Matsubara frequencies for fermions and

$$\tilde{S}^{-1}(p^0, \vec{p}) = \begin{pmatrix} p - \hat{m} - \mu\gamma_0 + gA^0\gamma_0 & 0 \\ 0 & p - \hat{m} + \mu\gamma_0 - gA^0\gamma_0 \end{pmatrix}.$$

The constituent quark mass is defined as

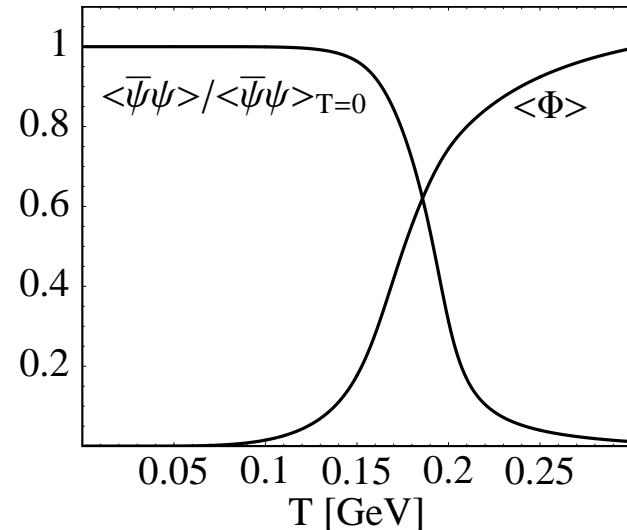
$$m = m_0 - \langle \sigma \rangle = m_0 - G \langle \bar{\psi}\psi \rangle.$$

Final form of  $\Omega$ :

$$\begin{aligned} \Omega(T, \mu) &= V(\Phi, T) + \frac{\sigma^2}{2G} - 2N_f T \int \frac{d^3 p}{(2\pi)^3} \left\{ \text{Tr}_c \ln \left[ 1 + \textcolor{red}{L} e^{-(E_p - \mu)/T} \right] \right. \\ &\quad \left. + \text{Tr}_c \ln \left[ 1 + \textcolor{red}{L}^\dagger e^{-(E_p + \mu)/T} \right] + 3 \frac{E_p}{T} \right\} \end{aligned}$$

with  $\text{Tr}_c \textcolor{red}{L} = \Phi$ ,  $\text{Tr}_c \textcolor{red}{L}^\dagger = \Phi^*$ .

$\mu = 0$  results

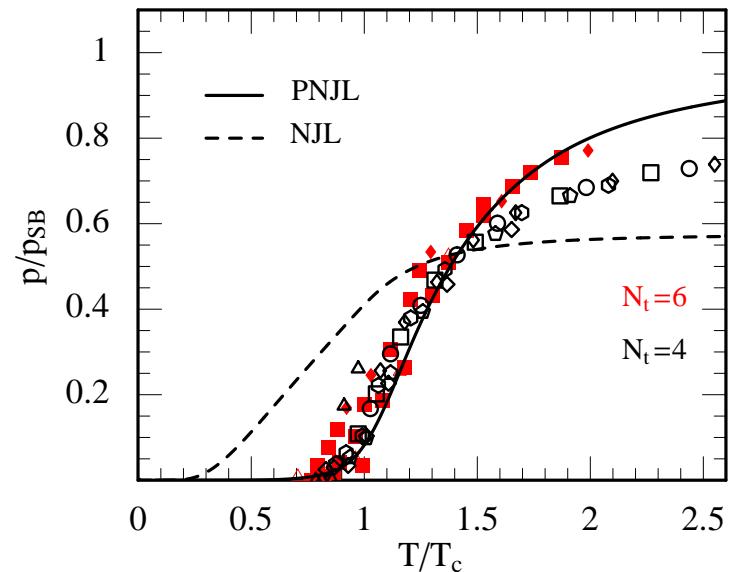


CHIRAL and  
DECONFINEMENT  
transitions  
almost coincide!

- ❖ Scaled pressure as a function of  $T/T_c$

$$\frac{p(T, \mu = 0)}{T^4} = -\frac{\Omega(T, \mu = 0)}{T^4}$$

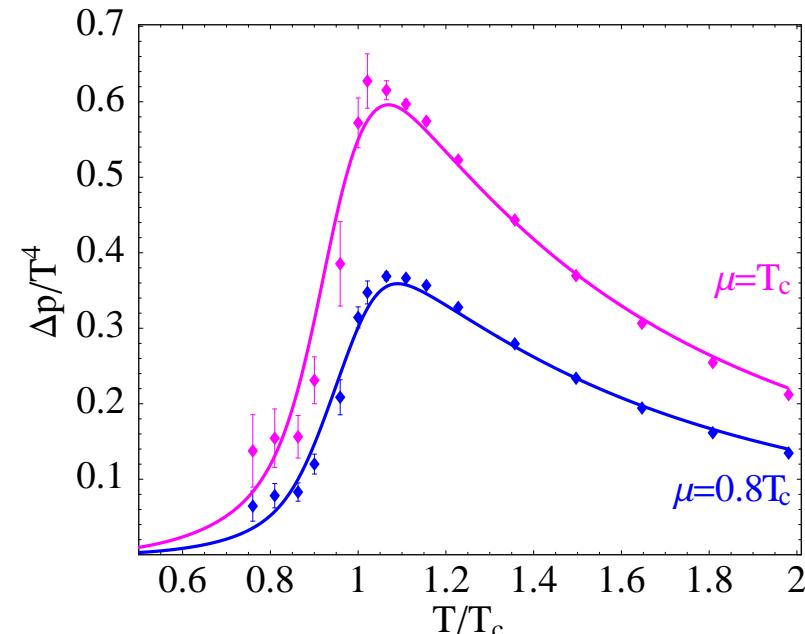
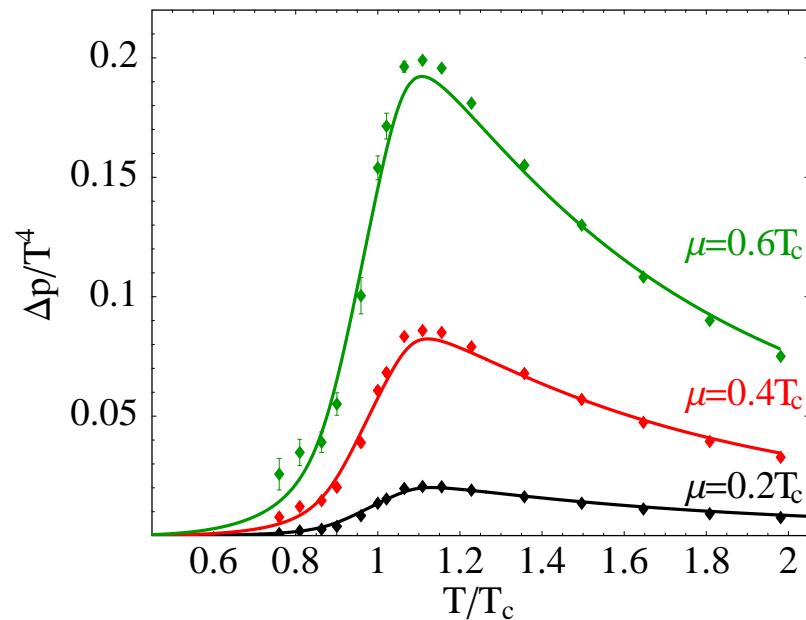
- ❖ Comparison with lattice data from  
**CP-PACS collaboration (2001)**



## Finite $\mu$ lattice data: pressure difference

- ❖ Scaled pressure difference as a function of  $T/T_c$

$$\frac{\Delta p(T, \mu)}{T^4} = \frac{p(T, \mu) - p(T, \mu = 0)}{T^4}$$

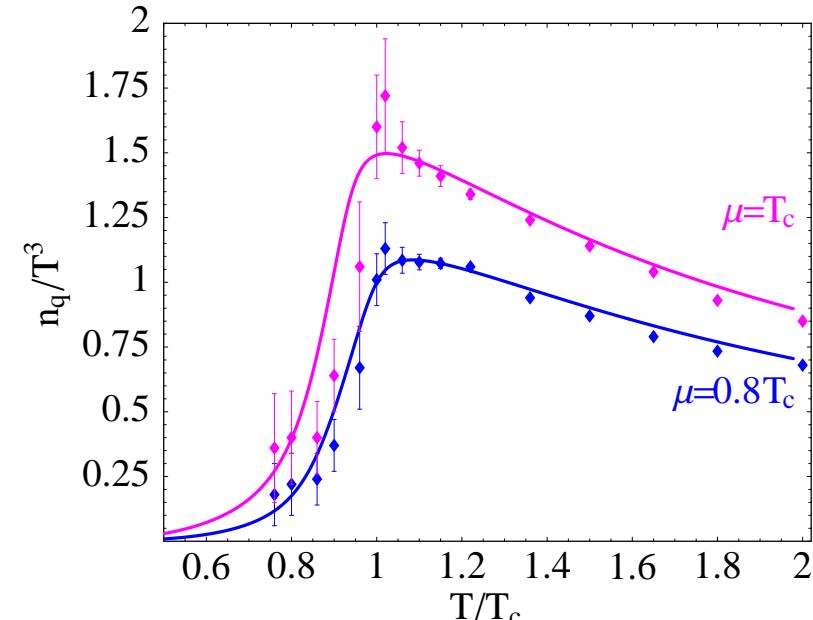
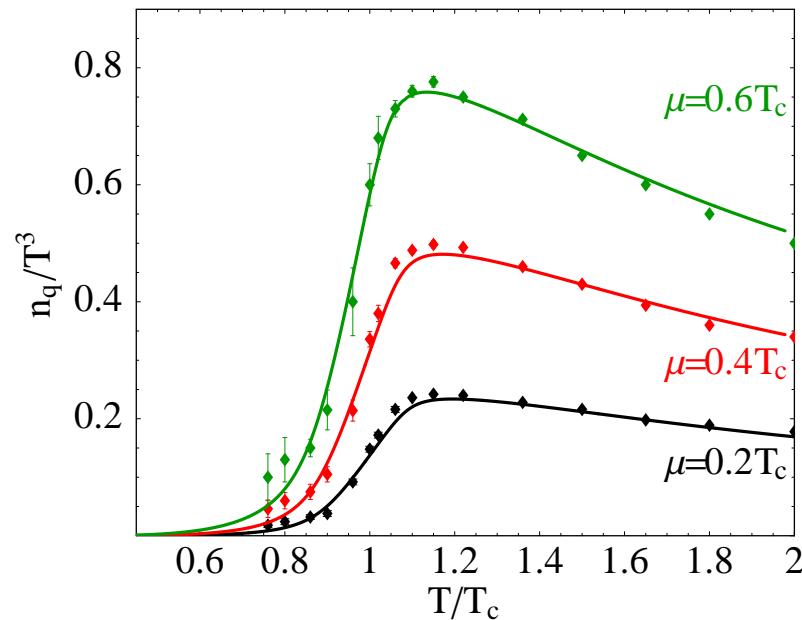


Scaled pressure difference as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from [Allton et al., PRD 68 \(2003\)](#).

## Finite $\mu$ lattice data: quark number density

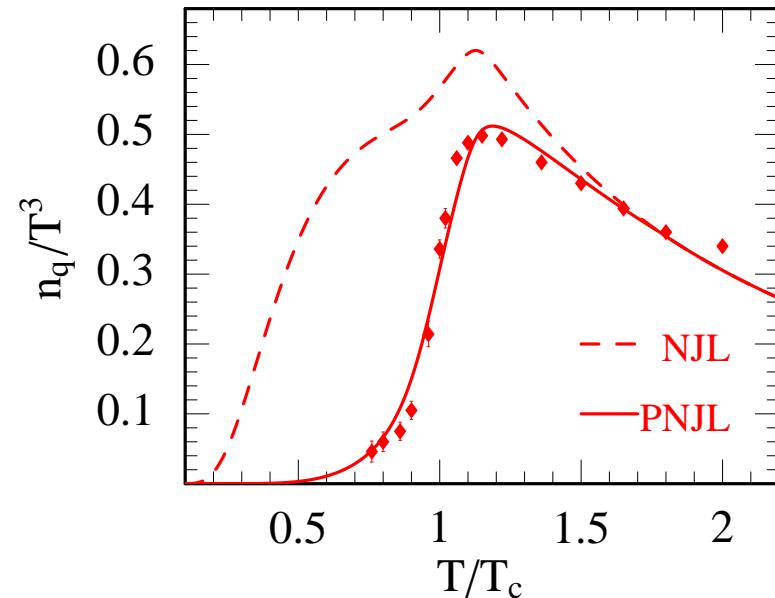
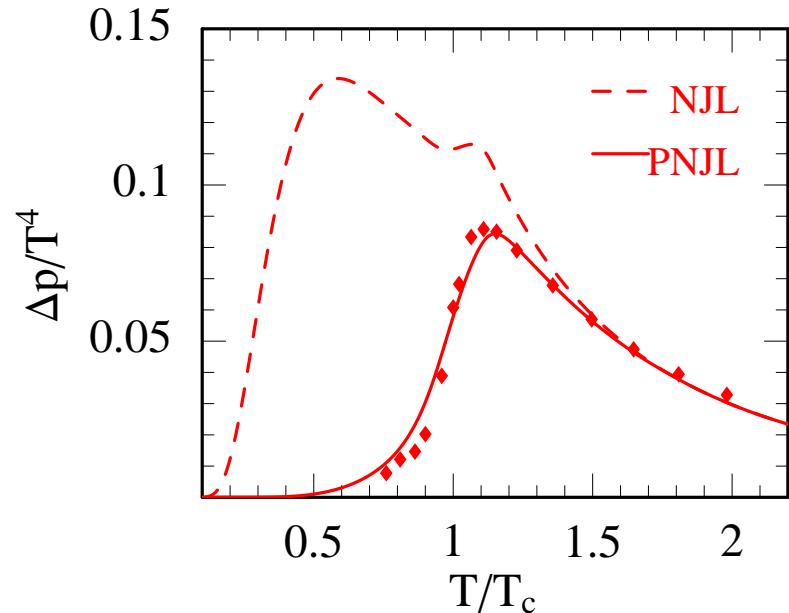
- ❖ Scaled quark number density as a function of  $T/T_c$

$$\frac{n_q(T, \mu)}{T^3} = -\frac{1}{T^3} \frac{\partial \Omega(T, \mu)}{\partial \mu}$$



Scaled quark number density as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from Allton *et al.*, PRD 68 (2003).

## Finite $\mu$ results in the standard NJL model

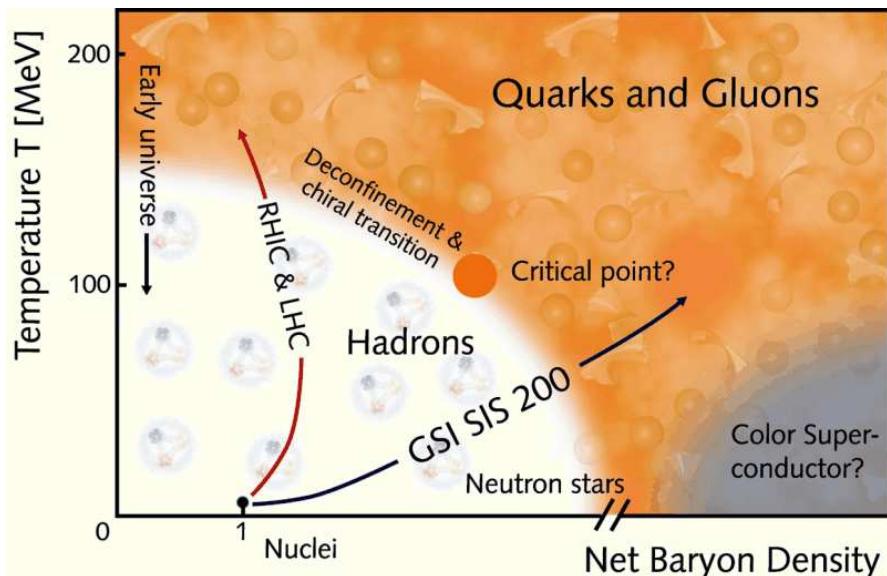


- ❖ Comparison between PNJL and standard NJL model results at [finite  \$\mu\$](#)
- ❖ NJL model alone fails in reproducing lattice data
- ❖ Incorporation of confinement **crucial** to reproduce lattice results

## Conclusions

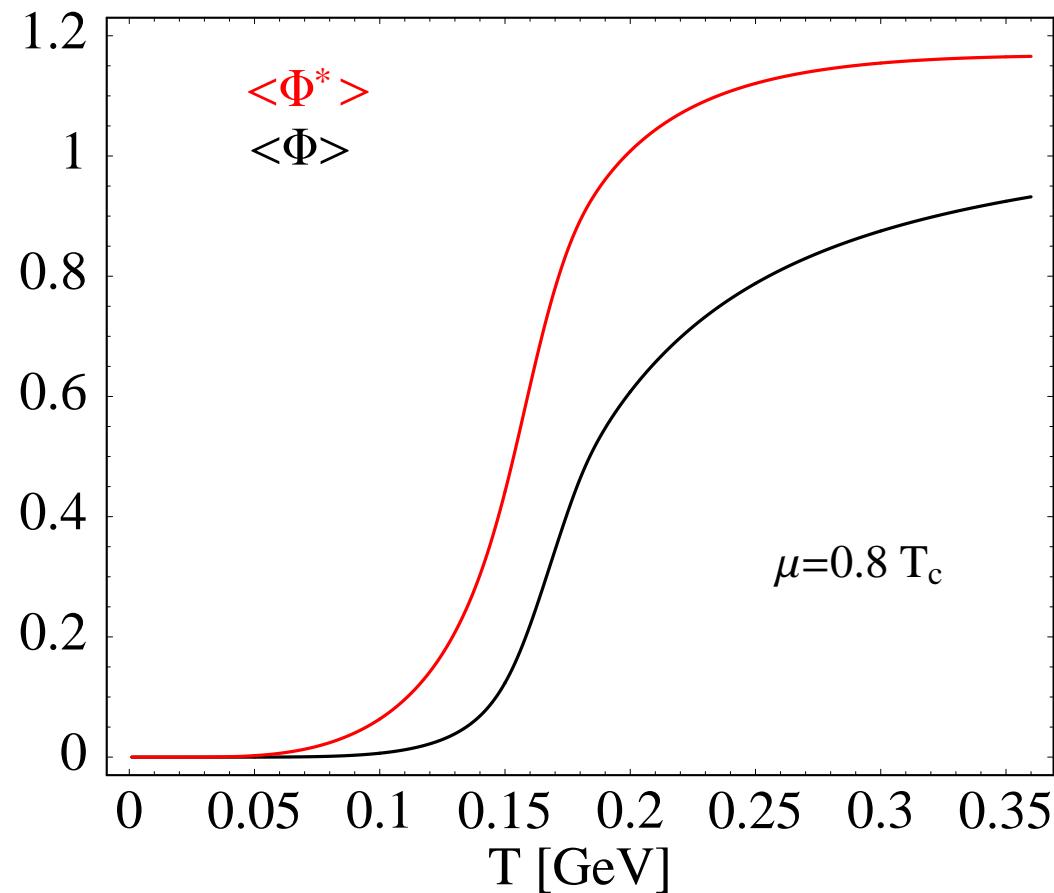
- ❖ A model which incorporates only chiral symmetry breaking fails in reproducing three-colour QCD thermodynamics
- ❖ PNJL model as a minimal synthesis of confinement and chiral symmetry breaking
- ❖ A description of QCD thermodynamics with our simple model works very well

## Outlook



- ❖ Lattice QCD can reach only moderate  $\mu$ 
  - ➡ exploration of high  $\mu$  regions
- ❖ Colour superconductivity cannot yet be studied on the lattice
  - ➡ introduction of diquarks
- ❖  $N_f = 2 + 1$

Backup slides

Finite  $\mu$  results

## Fixing the parameters

- ❖ In the Polyakov loop potential there are **7** parameters:
  - ➡  $a_0, a_1, a_2, a_3, b_3, b_4, T_0$
  - but **only 3** are free
  
- ❖ There are 4 constraints:
  - ➡  $T_0$  is fixed to **270 MeV**, the known critical temperature for the pure gauge system
  - ➡ The Polyakov loop must tend to **1** as  $T \rightarrow \infty$
  - ➡  $p(T) = -V(\Phi(T), T)$  tends to the ideal gas limit as  $T \rightarrow \infty$
  - ➡ At  $T = T_0$  the absolute minimum of  $V(\Phi, T)$  must jump from  $\Phi = 0$  to a **finite  $\Phi$**
  
- ❖ The remaining three parameters are fixed to reproduce the pure gauge lattice data

## Parameter fixing

We have three free parameters in the model:  $m_0$ ,  $\Lambda$ ,  $G$ . They are fixed by:

- ❖ The pion decay constant  $f_\pi$  is evaluated in the NJL model through the following relation:

$$f_\pi^2 = 4m^2 I_\Lambda^{(1)}(m) \quad \text{where} \quad I_\Lambda^{(1)}(m) = -iN_c \int \frac{d^4 p}{(2\pi)^4} \frac{\theta(\Lambda^2 - \vec{p}^2)}{(p^2 - m^2 + i\epsilon)^2}.$$

The empirical value is  $f_\pi = 92.4$  MeV.

- ❖ The quark condensate becomes

$$\langle \bar{\psi}_u \psi_u \rangle = -4m I_\Lambda^{(0)}(m) \quad \text{with} \quad I_\Lambda^{(0)}(m) = iN_c \int \frac{d^4 p}{(2\pi)^4} \frac{\theta(\Lambda^2 - \vec{p}^2)}{p^2 - m^2 + i\epsilon}.$$

Its “empirical” value derived from QCD sum rules is

$$\langle \bar{\psi}_u \psi_u \rangle^{1/3} \simeq \langle \bar{\psi}_d \psi_d \rangle^{1/3} = -(240 \pm 20) \text{ MeV}.$$

- ❖ The current quark mass  $m_0$  is fixed from the Gell-Mann, Oakes, Renner (GMOR) relation:

$$m_\pi^2 = \frac{-m_0 \langle \bar{\psi} \psi \rangle}{f_\pi^2}.$$

## Thermodynamic potential

The final form of the thermodynamic potential is

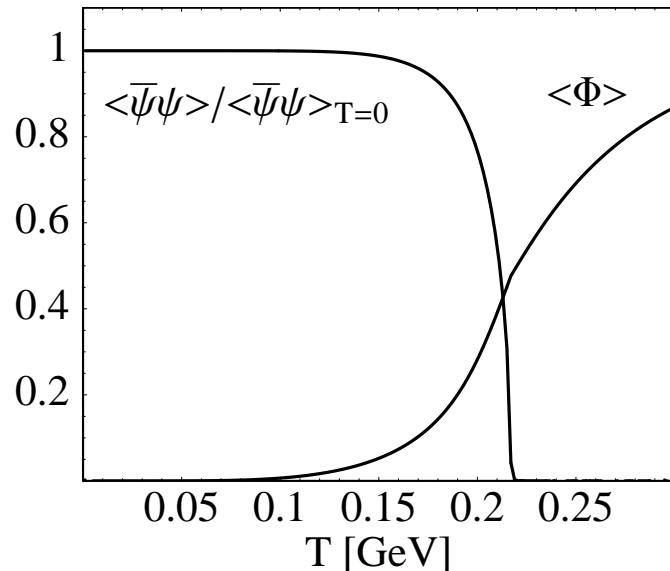
$$\begin{aligned} \Omega(T, \mu, \sigma, \Phi, \Phi^*) &= V(\Phi, \Phi^*, T) + \frac{\sigma^2}{2G} \\ -2N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left\{ E_p + \frac{T}{3} \left[ \ln \left[ 1 + 3 \left( \Phi + \Phi^* e^{-(E_p - \mu)/T} \right) e^{-(E_p - \mu)/T} + e^{-3(E_p - \mu)/T} \right] \right. \right. \\ &\quad \left. \left. + \ln \left[ 1 + 3 \left( \Phi^* + \Phi e^{-(E_p + \mu)/T} \right) e^{-(E_p + \mu)/T} + e^{-3(E_p + \mu)/T} \right] \right] \right\} \end{aligned}$$

with  $E_p = \sqrt{p^2 + m^2} = \sqrt{p^2 + (m_0 - \sigma)^2}$ .

Field equations:

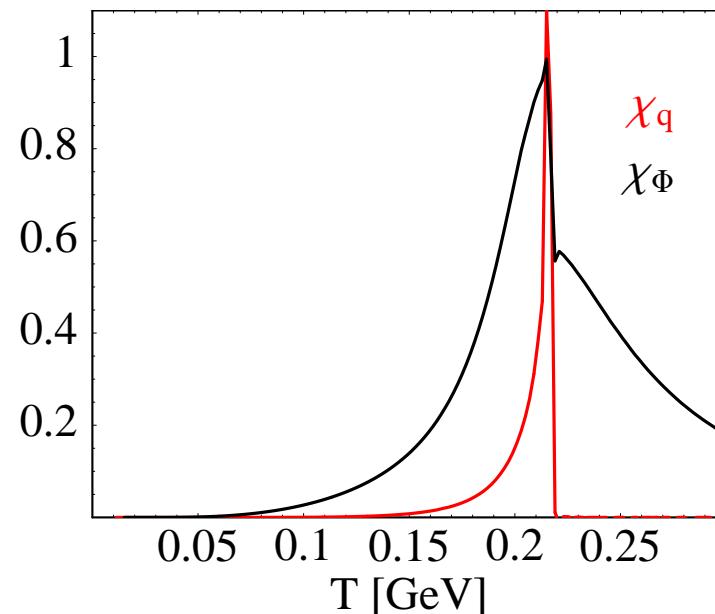
$$\frac{\partial \Omega(T, \mu, \sigma, \Phi, \Phi^*)}{\partial \sigma} = 0, \quad \frac{\partial \Omega(T, \mu, \sigma, \Phi, \Phi^*)}{\partial \Phi} = 0, \quad \frac{\partial \Omega(T, \mu, \sigma, \Phi, \Phi^*)}{\partial \Phi^*} = 0$$

## Confinement and chiral symmetry breaking: chiral limit ( $m_0 = 0$ )

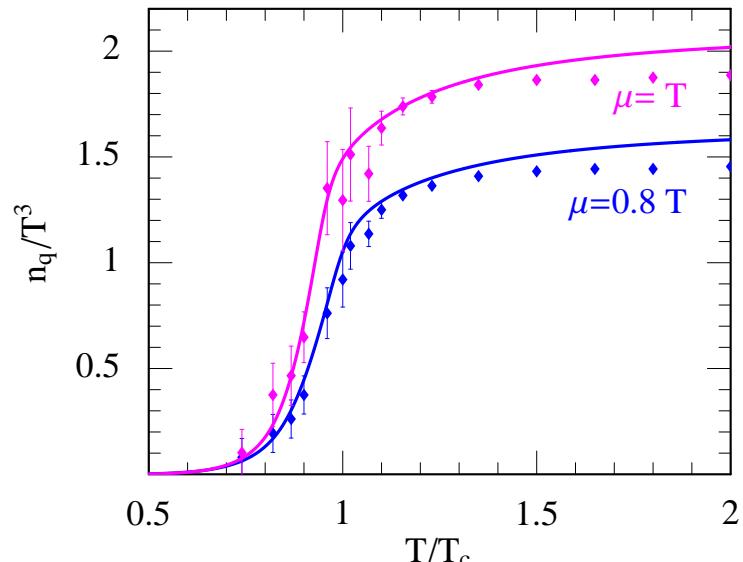
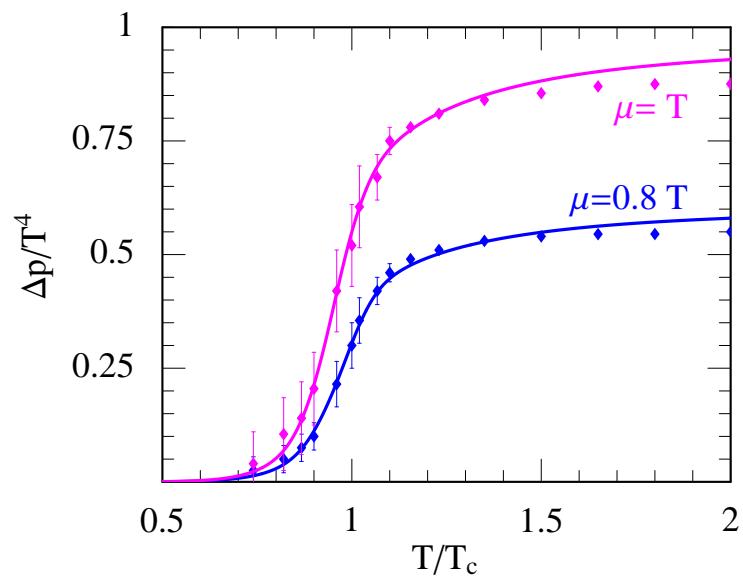
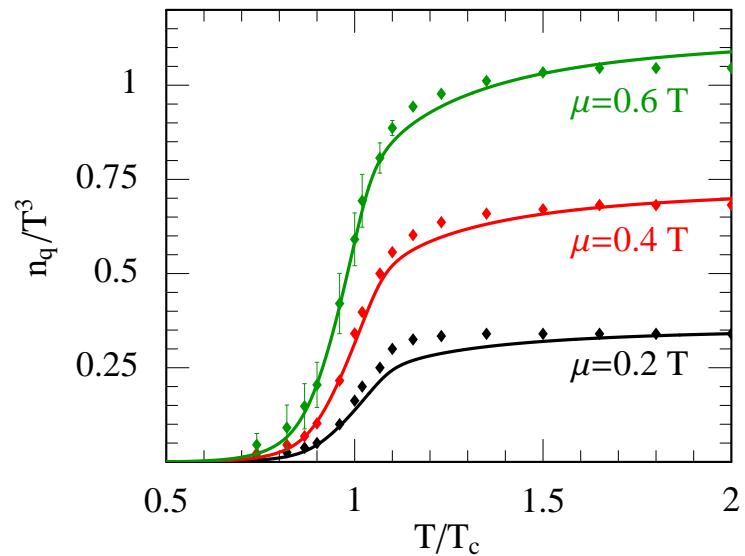
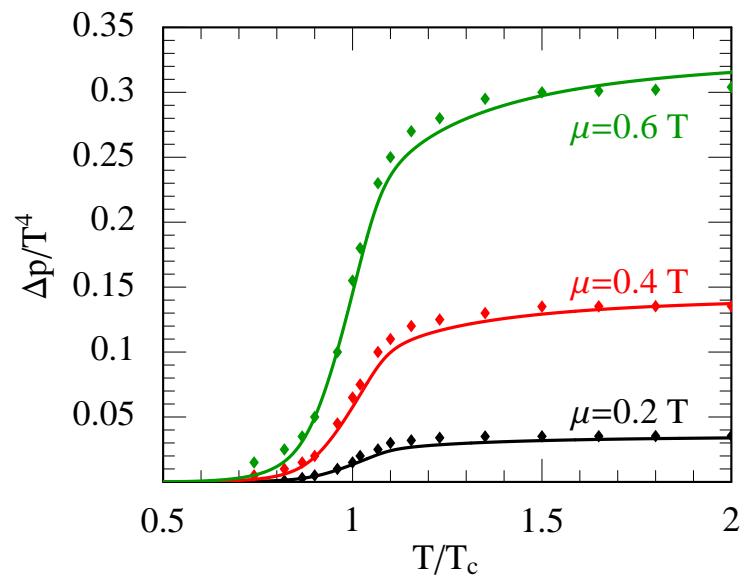


CHIRAL and DECONFINEMENT  
transitions coincide in the  
CHIRAL LIMIT!

$T_c \simeq 270$  MeV in pure gauge  
 $\Downarrow$   
 $T_c \simeq 220$  MeV with quarks

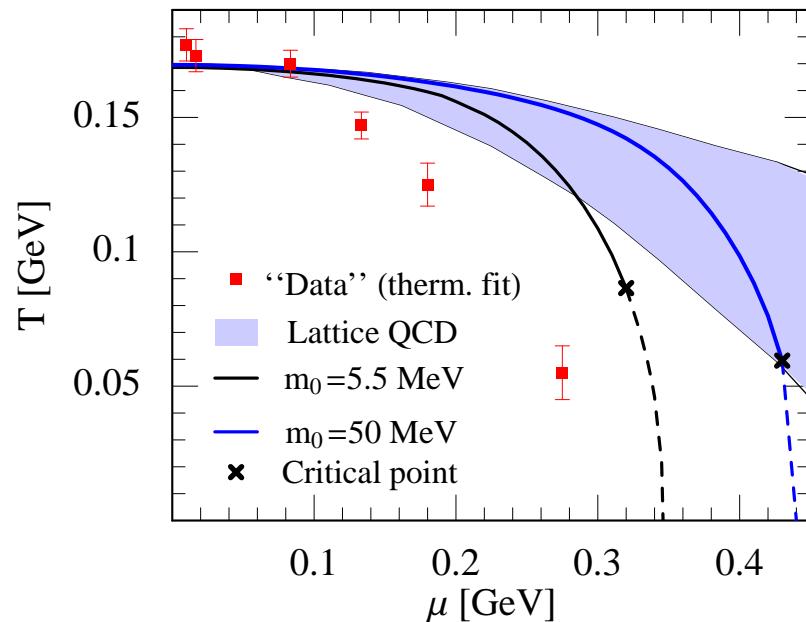


Going to higher  $\mu$ ...



Allton et al., PRD71 (2005)

# Towards the PHASE DIAGRAM



Therm. fit data from Andronic and Braun-Munzinger (2004)

First order transition  
at large chemical potential?

